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Optimal Continuous Thrust Orbit Transfer Using Evolutionary Algorithms

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Introduction

LOW-THRUST spacecraft gradually change their orbital elements (semimajor axis, inclination, and eccentricity), with the resulting trajectory resembling a three-dimensional spiral. A common goal for studies of low-thrust trajectories is to find the minimum-time or minimum-fuel paths. In general, these optimization problems are difficult to solve due to the long transfer time, eclipsing multirevolution transfers and sensitivity of a trajectory solution to boundary conditions.[1]

Background

Researchers have used many different optimization methods to attack optimal orbit transfer problems. The optimization methods used can be categorized into two classes: "direct" and "indirect." Direct methods solve the optimal control problem by adjusting the control variables at each iteration to reduce the performance index continually. Continuous control functions and the state differential equations are often parameterized, and the associated nonlinear programming problem may be solved with a gradient-based parameter optimization method. Indirect methods, on the other hand, use the calculus of variations to obtain a set of necessary conditions for a local optimum solution for the objective function. The resulting two-point boundary-value problem is very hard to solve for some problems due to its sensitivity to the initial guess of the costate variables. However, an advantage of indirect methods is that once the solution of the two-point boundary-value problem is obtained, the resulting trajectory is usually optimal.

Additionally, the global optimal solution is surrounded by many local optima, and most traditional optimization algorithms such as Newton's method and the method of steepest descent are local optimization algorithms. These seek only a local solution, a point at which the objective function is smaller than all other feasible points in its vicinity. To find a global solution, some type of global optimization approach should be used.

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In this Note, a simplified technique for searching for the initial costate values using evolutionary strategies (ESs) is presented. ESs are types of evolutionary algorithms (EAs) that have been developed and used to obtain optimal solutions in many fields. EAs have recently been attracting a great deal of attention as global optimization algorithms.[2,3] The main advantage of using EAs is that derivative information is not needed, and, thus, the objective functions are not required to be continuous over the search domain. Moreover, EAs' search points are spread out over the continuum of optimal solutions, unlike other optimization methods.

EAs are inspired from the Darwinian theory of evolution, which suggests that organisms have an ability to adapt themselves to the environment over many generations. More precisely, they evolve by changing their genetic information through mating, recombination or crossover, and mutation. After many generations have passed, only certain traits survive. Igarashi[4] states that ESs are superior to the genetic algorithms (GAs) from the perspective of numerical precision problems. The binary representation used in the GAs limits further improvement of the solutions. Thus, ESs should be used for real-valued vector optimization problems, which require adequate accuracy on the solutions. However, the performance of GAs is not always inferior to ESs for all applications. In most applications, the choice of what kind of EAs is used is left entirely to the user.

The objective function for ESs measures how close the final orbit conditions achieved by the solutions obtained from the ESs are to the desired conditions. We use equinoctial orbital elements[5] a, h, k, p, q, and L to avoid singularities in circular, equatorial orbits. The full set of governing equations in equinoctial elements for a continuous constant acceleration f_t are shown in Ref. 5 as

$$\dot{\boldsymbol{x}}_e = f_t \boldsymbol{M}\boldsymbol{u} + n\boldsymbol{e}_L \tag{1}$$

where the orbital mean motion n is given by $n = \sqrt{\mu/a^3}$, μ is the gravitational constant, $\boldsymbol{M}$ is a 6×3 matrix relating the time rate of change of the orbital elements to the thrust vector, and $\boldsymbol{u}$ is a unit vector in the thrust direction,

$$\boldsymbol{u} = u_f \hat{f} + u_g \hat{g} + u_w \hat{w}, \qquad \|\boldsymbol{u}\| = 1 \tag{2}$$

in the $\hat{f}$, $\hat{g}$, and $\hat{w}$ equinoctial frame. The partial derivatives of the equinoctial elements with respect to the position and velocity vectors, $\boldsymbol{r}$ and $\dot{\boldsymbol{r}}$ are also presented by Kechichian.[5]

For the orbit transfer problem with continuous constant thrust acceleration, both the optimal minimum-time transfer and the minimum-fuel transfer occur simultaneously. Thus, the performance index to be minimized is the final time:

$$J = t_{\text{final}} \tag{3}$$

subject to the equations of motion with position and velocity vector quantities known at time $t = 0$ and at some unknown final time $t = t_{\text{final}}$ where t_{final} is the time duration of the transfer.

A scalar function H, the Hamiltonian (see Ref. 1), for this system is

$$H = \boldsymbol{\lambda}^T \boldsymbol{M}\boldsymbol{u} f_t + \lambda_L n = \boldsymbol{\lambda}^T \dot{\boldsymbol{x}}_e \tag{4}$$

where $\boldsymbol{\lambda} = (\lambda_a \;\; \lambda_h \;\; \lambda_k \;\; \lambda_p \;\; \lambda_q \;\; \lambda_L)^T$ is the vector of Lagrange multipliers adjoint to the state variable $\boldsymbol{x}_e$. The partial derivatives of $\boldsymbol{M}$ are found in Refs. 4 and 5. The Euler–Lagrange or adjoint differential equations are given by

$$\dot{\boldsymbol{\lambda}} = -\frac{\partial H}{\partial \boldsymbol{x}_e} = -\boldsymbol{\lambda}^T \frac{\partial \boldsymbol{M}}{\partial \boldsymbol{x}_e} f_t \boldsymbol{u} - \lambda_L \frac{\partial n}{\partial \boldsymbol{x}_e} \tag{5}$$

For a fixed time t_{initial}, the minimization of t_{final} gives rise to the transversality condition $\lambda_{L,\text{target}} = 0$ and $H_{\text{target}} = 1$ (Refs. 1 and 6). Under the assumption of a constant acceleration, the solution of this problem results in the time history of the thrust direction that describes the orbital trajectory, which is performed by selecting $\boldsymbol{u}$ parallel to $\boldsymbol{M}^T \boldsymbol{\lambda}$ for all times. Thus, the thrust direction is given by

$$\boldsymbol{u} = \boldsymbol{M}^T \boldsymbol{\lambda} / \|\boldsymbol{M}^T \boldsymbol{\lambda}\| \tag{6}$$

When the in-plane thrust angle is defined as α and the out-of-plane thrust angle is defined as β, this thrust vector is expressed in these angles by

$$u_f = \cos\alpha \cos\beta \tag{7}$$

$$u_g = \sin\alpha \cos\beta \tag{8}$$

$$u_w = \sin\beta \tag{9}$$

Equations (1) and (5) are now integrated simultaneously with a seventh-order variable step Runge–Kutta–Fehlberg RK78 integrator, with the tolerance 10^{-9} from the initial time to the final time.

For the optimal transfer problem addressed in this Note, the initial and final orbit properties are known, and the initial values of the Lagrange multipliers $\boldsymbol{\lambda}_{\text{initial}} = (\lambda_a \;\lambda_h \;\lambda_k \;\lambda_p \;\lambda_q \;\lambda_L)^T_{\text{initial}}$ and the transfer time t_{final} are unknown. Thus, the vector components of all individuals that make up the population of the ESs consist of these seven unknowns, and these are optimized so that the final conditions are satisfied.

To apply the EAs to the problem, an objective function must be defined. Because the goal is to find the optimal initial set of Lagrange multipliers and transfer time such that the final equinoctial elements match the target conditions, the following objective function is defined:

$$\begin{aligned} f = {} & w_1(a - a_{\text{target}})^2 + w_2(h - h_{\text{target}})^2 + w_3(k - k_{\text{target}})^2 \\ & + w_4(p - p_{\text{target}})^2 + w_5(q - q_{\text{target}})^2 \\ & + w_6(\lambda_L - \lambda_{L\text{target}})^2 + w_7(H - H_{\text{target}})^2 \end{aligned} \tag{10}$$

where the final equinoctial elements from the integration and the target conditions are matched. The variables $\boldsymbol{w} = (w_1 \; w_2 \; w_3 \; w_4 \; w_5 \; w_6 \; w_7)$ are weights that can be adjusted to adjust the convergence of some elements. The ESs try to minimize this objective function and, as a result, minimize the performance index.

Results

A representative low-Earth-orbit (LEO) to geostationary-Earth-orbit (GEO) transfer problem with constant continuous thrust acceleration is examined. The optimal orbit transfer is achieved by means of the ESs, and the results obtained from this analysis are compared to previously published results.[1] The departing position on the initial orbit is fixed, and the arrival position is unconstrained. The constant thrust acceleration is set to $f_t = 9.8 \times 10^{-5}$ km/s^2.

Igarashi[4] found the appropriate population size for this analysis, based on a numerical experiment in his thesis. He also developed the objective function used in this Note. Finally, the searching boundary constraints, defined in Ref. 1, permit the ESs to perform appropriately. Another numerical experiment was conducted in Ref. 4 that examined the effects of the population size on the ESs convergence characteristics. Overall, trial and error showed that the population size for this analysis should be set somewhere between 10 and 20. Generally, a wider searching boundary translates into the need for a larger population size. In the objective function, Eq. (10), the weighting parameters $\boldsymbol{w}$ adjust the convergence of some elements relative to others. Because a_{target} is orders of magnitude higher than other elements in the objective function, the minimum objective function value is highly dependent on the first term $(a - a_{\text{target}})^2$, unless a modification is made to this term. To balance all terms in the objective function, absolute and relative errors to the target values are considered, and the final weights on the objective function are shown in Table 1.

Table 1 Weight used in objective function

Weight	Value
w_1	10^{-3}
w_2	10^6
w_3	10^{12}
w_4	10^6
w_5	10^{11}
w_6	10^6
w_7	10^6

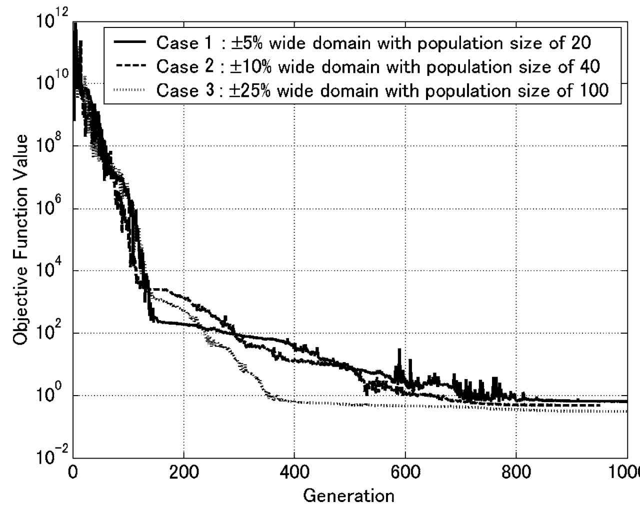

Fig. 1 Objective function value comparison with various width searching domains.

Fig. 2 Standard deviation comparison with various width searching domains.

We examined three cases: ±5% (case 1), ±10% (case 2), and ±25% (case 3). Based on the analysis in Ref. 3, the population size of 20 is used for case 1, 40 for case 2, and 100 for case 3. This is reasonable because case 3 uses a searching domain that is five times wider than that of case 1. The computational results of the ESs' performances of each case are plotted in Figs. 1 and 2.

Figure 1 shows the objective function values for each case and how the ESs' solution approaches the optima from Ref. 1. Note that the slopes of the first 150 generations are very steep. This is because all initial populations in the ESs are created randomly and start with poor initial guesses. In Fig. 1, all cases have different terminal converged solutions, that is, the solutions obtained from both cases 1 and 2 are clearly realized as local optima because the solution of case 3 is smaller. The ability of ESs to solve this type of problem is, thus, demonstrated, but not rigorously validated.

Figure 2 shows the convergence behaviors of the population of the ESs for each case. The standard deviations of the objective values of all of the individuals are directly affected by mutation. Thus, the shape of Fig. 2 is much more uneven than that of Fig. 1. However, the standard deviation values are gradually moving toward zero, which indicates convergence to a single solution. For case 1, the generation at which the standard deviation becomes less than 10^{-1} is 931 and 905 for case 2 and 929 for case 3. It is probably coincidental that these three cases converge to the optima at the almost same number of generations, and, moreover, the shapes of the plots of the three cases in Fig. 2 are nearly overlapping most of the time. Thus, there appears to be a proportional relationship between the extent of the searching domain and the population size needed for the appropriate ESs' performance or the number of function evaluations. In fact, the total computation times for these three cases are proportional to the size of the population or the width of the searching area. The simulations were performed on a personal computer with an Intel® Pentium® 4 processor, operating with a clock speed of 3.00 GHz and 512 MB RAM. The software was written in MATLAB®. The total computation times of this optimization problem are approximately 4 h for case 1, 8 h for case 2, and 20 h for case 3.

The integration was carried out using these adjoint variables toward to the optimized transfer time. The final set of equinoctial orbital elements and the time histories of the adjoint variables found using the ES approach are indistinguishable from Kechichian's results.[1]

Conclusions

The use of ESs for an optimal orbit transfer problem from LEO to GEO is achieved using various searching domain widths. The results in this Note are comparable to previously published results, given the restricted width of the searching boundary constraints that were used to choose the optimization parameters. A wider searching domain needs a larger number of function evaluations, and the total computation time is substantially longer. For problems where the initial costate variables are completely unknown, a very wide initial search domain can be implemented, but the use of the parallel computing is recommended to reduce the additional computational burden.

We also note that the difficulty of this analysis is not only choosing the searching boundaries, but also in selecting the weights of the objective function. Different objective functions produce different optimal objective function values; thus, the optimal solution generated by the ESs will differ. In this Note, relative error methods are used to determine these weights, and more accurate investigation would be necessary.

We conclude that ESs appear to be capable of finding initial sets of adjoint variables, although they do not always find the global solution due to the randomized process in the algorithms. However, these algorithms are quite robust in searching and finding the optimal initial adjoint variables.

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